

## Social Choice and Individual Liberties

### I. Introduction and Review

... a stunning principal of social organisation <sup>1</sup>

Some subset of choices belong in the private or protected sphere of the individual. This idea is deeply imbedded in the culture, values and institutions of Western Civilization. It seems to qualify as a stunning principle of social organization. Sen (1970) challenged us to rethink this private sphere, in the context of its relation to the Pareto principle. Unanimity is another stunning principle of social organization. If everyone prefers x to y, society should prefer x to y.

Nonetheless, Sen formalized an extremely weak version of liberalism (or libertarianism as he later called it) and found it inconsistent with a weak form of the Pareto principle together with an unrestricted domain of social choice. An even more disturbing consequence emerged from Sen (1976). There he demonstrated the *Paretian epidemic*.<sup>2</sup> If one person is made decisive over one pair, representing a tiny protected sphere of personal choice, then by virtue of the Paretian epidemic he is potentially semi-decisive over *every pair* of social states. So no one can be decisive over any pair whatsoever, thereby ruling out the possibility of guaranteeing the libertarian rights of anyone else. <sup>3</sup>

The objective of this paper is to examine the formalization of the private or protected sphere used by Sen. His formalization is viewed from a number of perspectives. The analysis

---

I would like to thank Professor Charles Rowley, Director of the Locke Institute, Professor of Economics at George Mason University and co-editor of *Public Choice*. He introduced me to the topic and graciously agreed to sponsor a directed reading on the subject. After reading the initial draft, he directed me immediately to deJasay (1996). deJasay had already worked through those aspects of classical liberalism I was only just beginning to understand. The resulting paper is much clearer and to the point.

<sup>1</sup> Schelling (1984 p. 99)

<sup>2</sup> This consequence follows from the acceptance of the Pareto principal and Condition U in conjunction with Condition L.

leads to a re-formalization of Condition L. The consequences of that reformulation are explored over a range of topics in light of a newly developed view of libertarian ideals. Those topics include the role of mutual consent, the liberal partition, Gibbard's paradox, unanimity, and the relevance of both the impossibility of a Paretian liberal and the Paretian epidemic.

It has been some time since these important articles were first published. Sen's work follows in the tradition of Arrow (1950) which is referred to as Social Choice Theory. Arrow produced an impossibility theorem using four conditions and two axioms. No social welfare function can be formulated which has the properties of connectedness and transitivity (the axioms) which does not violate one of the four conditions.<sup>4</sup> Sen weakened the requirements. He dropped two of Arrow's initial conditions and added Condition L.<sup>5</sup> He further reduced the demands on the collective decision function, only requiring that it produce a best alternative. This *social decision function* required neither transitivity of strict preference nor transitivity of indifference.

He introduces his Impossibility of a Paretian Liberal by noting that a common objection to the method of majority decision is that it is illiberal."<sup>6</sup> His objective in that paper was to demonstrate that Pareto optimality is illiberal as well.<sup>7</sup> He formalized the notion that, Given other things in society, if you prefer to have pink walls rather than white, then society should permit you to have this, even if a majority of the community would like to see your walls white. <sup>8</sup> From the specific to the general, then, we have:

---

<sup>3</sup> Sen (1976 p. 220)

<sup>4</sup> Arrow, K. (1951) The four conditions were Conditions U (Universal Domain of Social Choice), P (Pareto principal), D (Non-dictatorship) and I (Independence of Irrelevant Alternatives).

<sup>5</sup> The dropped conditions were D and I.

<sup>6</sup> Sen (1970 p.152)

<sup>7</sup> The weak Pareto principal was to serve, apparently, as a proxy for this demonstration. The connection between the Pareto principal and the concept of Pareto optimality in welfare economics is by no means obvious. This is not addressed in this paper.

<sup>8</sup> Sen (1970 p.152)

Condition U (Unrestricted Domain). Every logically possible set of individual orderings is included in the domain of the collective choice rule.

Condition P If every individual prefers any alternative  $x$  to another alternative  $y$ , then society must prefer  $x$  to  $y$

Condition L: For each individual  $i$ , there is at least one pair of alternatives, say  $(x, y)$ , such that if this individual prefers  $x$  to  $y$ , then society should prefer  $x$  to  $y$ , and if this individual prefers  $y$  to  $x$ , then society should prefer  $y$  to  $x$ .

Theorem I. There is no social decision function that can simultaneously satisfy Conditions U, P and L.<sup>9</sup>

Theorem II, below, subsumes Theorem I since Condition  $L^*$  was a weaker version of libertarianism than L.

Condition  $L^*$ : There are at least two individuals such that for each of them there is at least one pair of alternatives over which he is decisive, that is, there is a pair of  $x, y$  such that if he prefers  $x$  (respectively  $y$ ) to  $y$  (respectively  $x$ ), then society prefers  $x$  (respectively  $y$ ) to  $y$  (respectively  $x$ ).

Theorem II. There is no social decision function that can simultaneously satisfy Conditions U, P and  $L^*$ .<sup>10</sup>

Sen used the method of proof by cases to establish the inconsistency of these conditions. In the next section we examine the notions of decisiveness and private choice used by Sen in his analysis.

### **I. Privately decisive choices.**

The most basic of all decisions is *who* shall decide.<sup>11</sup>

---

<sup>9</sup> Sen (1970 p. 153)

<sup>10</sup> Sen (1970 p. 154)

<sup>11</sup> Sowell (1980 p.40)

There is no doubt that the libertarian claim requires that there exist at least one pair for which each individual may be decisive, irrespective of the preferences of all others.<sup>12</sup> This is an essential aspect of liberal thought. In the language of logic, the claim to which Sen refers is an *existential* claim. The claim is universal with respect to individuals but existential with respect to pairs. Sen elected to characterize this claim as a determination of the social preference ordering by each individual with respect to that pair. Symbolically,

$$(1.1) \forall i(\exists(x, y) : xP_i y \Rightarrow xPy \text{ and } yP_i x \Rightarrow xPy).^{13}$$

There would be pairs for which the claim would be true and pairs for which the claim would be false. All that remains is to *specify* for which pairs the claim holds. Note that if it were true for all pairs it would be a *universal* claim with respect to pairs as well.

Alternatively, Condition L can be thought of as a modified Condition D. Recall from Arrow's analysis Condition D paraphrased below.<sup>14</sup>

Condition D: A social welfare function is said to be dictatorial if there exists an individual  $i$  such that for all  $x$  and  $y$ ,  $xP_i y$  implies  $xPy$  irrespective of the preferences of all others.

In this case, however, it is desirable that each individual be decisive, irrespective of the preferences of all others. For some pairs, however, and not others. Alternatively, we could represent Condition L as Condition D\*.

Condition D\*: A social welfare function is said to be libertarian, if for all  $i$  there exists some  $x$  and  $y$ , such that  $xP_i y$  implies  $xPy$  and  $yP_i x$  implies  $yPx$ , irrespective of the preferences of all others.

---

<sup>12</sup> The phrase *libertarian claim* was introduced in Gibbard (1974)

<sup>13</sup> In the analysis that follows the focus will be on Condition L rather than L\*. The problems that exist with Condition L are not resolved by moving to L\*. They are resolved by correcting L. Condition L\* is highly illiberal, in some ways the antithesis of libertarian thought.

<sup>14</sup> Condition D was actually that no social welfare function was to be dictatorial. A definition of dictatorial was given similar to the one above which we call Condition D.

Symbolically,

$$(1.2) \forall i(\exists(x,y): xP_iy \Rightarrow xPy \text{ and } yP_ix \Rightarrow xPy).$$

The representations 1.1 and 1.2 are symbolically equivalent. In either case, the concept is that each individual determines the social preference for some pair(s). Again, however, we need to specify which pairs we want each individual to be dictatorial over and which pairs we do not want them to be dictatorial over.

Therefore, the first problem with Condition L is that it authorizes every individual to be socially decisive over at least one *unspecified* pair. Condition L does not formalize the notion that the subject matter of the claim is only privately decisive choices.<sup>15</sup> Sen offered this qualification. The acceptability of Condition L will depend on the nature of the alternatives that are offered for choice, and if the choices are *all non-personal*, e.g. to outlaw untouchability or not, to declare war against another country or not, this condition should not have much appeal (Sen 1970a). But in choices involving personal variations of the kind discussed earlier, L would seem appealing.<sup>16</sup> (Italics added) The acceptability of L is based on whether *all* the elements of choice are decisively private. It is not sufficient that they are *not all non-personal* or even that *some* of the elements of choice are privately decisive. This essential distinction will be demonstrated below. As the Conditions are manipulated and the results interpreted, distinctions are overlooked by not formalizing this requirement. Formalizing this requirement leads to the discovery of important insights into the irrelevance of the impossibility result and the true nature of the libertarian claim.

---

<sup>15</sup> There are many aspects to libertarian thought. Sen's oft repeated statement that Condition L\* is a minimal requirement of all of them is inaccurate. For some types of choices the liberal desires that the individual be a dictator. For other types of choices the liberal requires mutual consent. Confusion about this distinction abounds in the literature.

<sup>16</sup> Sen (1976 p.218)

While the existential statement made in Condition L is certainly true, at least one pair is not sufficiently descriptive for the manipulations that follow. In the language of set theory, we need a *sentence*,  $S(X)$ , which specifies the *properties* of  $x$  and  $y$ , which distinguish those pairs for which an individual may be decisive from those for which they may not be decisive.<sup>17</sup> Let  $X$  be the set of all possible choices, and  $DPS_j$  be the decisive private sphere of  $j$ . The sentence  $S(X)$  would specify those properties and create a subset of  $X$ ;  $DPS_j$ . Presume that not all choices are decisively private. The union of each decisive private sphere would be a proper subset of  $X$ . This is consistent with the idea that the libertarian claim is not a universal claim pertaining to all  $x$  and  $y$ .

$$(1.3) \cup_i DPS_i \subset X$$

Condition L should be restated to specify the set of elements over which each individual can be decisive.

Symbolically,

$$(1.4) \forall i(x, y \in DPS_i \Rightarrow (xP_i y \Rightarrow xPy \text{ and } yP_i x \Rightarrow xPy)).$$

In words: A social welfare function is said to be libertarian if for each individual, if  $x$  and  $y$  are both elements of the decisive private sphere of that individual,  $xP_i y$  implies  $xPy$  and  $yP_i x$  implies  $yPx$ , irrespective of the preferences of all others.

Consider the existence of a rule that assigns elements of  $X$  to  $DPS_i$  for all  $i$ . The rule may be a complete list or a general principal. Symbolically,

$$(1.5) \forall i(\exists f_L: X \rightarrow DPS_i).$$

An implication of that rule is that for each  $i$ , the decisive private sphere of that individual is not empty.

---

<sup>17</sup> This concept is referred to the axiom of specification or the axiom of separation. A set is uniquely determined by those properties that distinguish it from the universal set from which the elements are selected.

$$(1.6) \forall i (DPS_i \neq \emptyset).$$

This is consistent with the existence claim. The implication, in fact, goes both ways.

$$(1.7) \forall i (\exists f_L: X \rightarrow DPS_i) \Leftrightarrow \forall i (DPS_i \neq \emptyset).$$

The rule would have to make a consistent assignment of elements for all  $i$ , such that if an element of the pair  $(x,y)$  was an element of  $DPS_j$ , then it would not be an element of  $DPS_k$ . Both  $j$  and  $k$  cannot be feasibly decisive over the same elements of choice. Consider this a restriction on  $f_L$ .

For example: Let  $x = \text{John sleeps on his back}$  and  $y = \text{John sleeps on his belly}$ , then  $x$  and  $y$  are elements of John's decisive private sphere and not also elements of Karl's decisive private sphere. If a choice is decisively private to John, it is not also decisively private to Karl.

Symbolically,

$$(1.8) \forall j,k (x,y \in DPS_j \Rightarrow x,y \notin DPS_k).$$

Equivalently, the decisive private sphere of each individual would be pairwise disjoint from every other.

$$(1.9) \forall j,k (DPS_j \cap DPS_k = \emptyset).$$

Gibbard (1974) and Farrell (1976) commented on the need for a consistent assignment of rights (liberties is the appropriate concept) but did not offer an approach. What properties would  $x$  and  $y$  have to be elements of  $DPS_i$  and be consistent with liberal thought? Let  $C_i$  denote the choice of action for  $i$  from elements of his decisive private sphere. From the libertarian perspective:

Condition MFL: (mutually feasible liberty) For all  $j$  and  $k$ ,

if  $x,y \in DPS_j$  and  $C_j(\{x,y\}) = x$  or  $y$  is feasible to  $j$ , then  $w,z \in DPS_k \Rightarrow C_k(\{w,z\}) = w$  or  $z$  is still feasible to  $k$ .

This is another restriction on  $f_L$ . The elements,  $x$  and  $y$ , of  $j$ 's decisive private choice may be mutually exclusive with respect to each other. They may *not* be mutually exclusive with respect to the elements of  $k$ 's decisive private choices,  $w$  or  $z$ . This is the second problem with Sen's approach. He presumes that the choices are mutually exclusive. For example: If John sleeps on his back or his belly, Karl may still sleep on either his back or his belly. In fact, Karl may still choose between reading or not reading *On Liberty*, living in New York or Massachusetts, etc., even though John decided to sleep on his back.

Also, as long as the choices are not mutually exclusive, each individual may have a multitude of decisive private choices. Suppose that  $x, y, w, z \in DPS_j$ , then  $C_j(\{x, y, w, z\}) = (x \text{ and } w)$  is feasible as long as the pairs  $x, y$  and  $w, z$  are not mutually exclusive with respect to each other. John may sleep on his back or his belly in New York or Massachusetts. If we add a time horizon, he may choose to sleep on his back on Tuesday, his belly on Thursday, read *Das Kapital* on Wednesday, etc. and all while living in New York rather than Massachusetts. Simultaneously, Karl may sleep on his belly, read *On Liberty*, etc. and do it all in El Paso rather than Tulsa. There is a multitude of elements over which John and Karl may be decisive. The choices from the decisive private spheres of all individuals are mutually feasible.

Sen moves too quickly from Condition L to  $L^*$  and in the wrong direction. He moves to limit the number of *individuals* to whom the liberty applies rather than limit the *alternatives* to which the liberty applies. His lead in to Condition  $L^*$  from L: Such freedom may not be given to all but to a subset of individuals. However, to make *sense* the subset must have more than one member, since if it includes only one then we might have a dictatorship. Hence we *demand* such freedom for at least two members.<sup>18</sup> (Italics added) To make *sense* we must specify the nature of the alternatives over which the individual may be decisive and have a consistent assignment of

liberties. To *demand* that the subset has two individuals is to demand too little. Co-liberties are feasibly co-existent because they pertain to social states which are not mutually exclusive across individuals. For each individual and decisiveness are both essential elements of liberal thought but if and only if all of the elements of choice are private to that individual.

At this point it is appropriate to examine the relevance of the first and third parts of Sen's impossibility result with respect to privately decisive choices. Remember that we are talking only about decisively private choices. The following sections will deal with another libertarian principal that applies to choices and actions for which the individual may not choose decisively; mutual consent.

Indeed, Condition L, as formulated by Sen, is not consistent with *itself* because it does not specify which elements each individual may be decisive over.

Proof:

- 1)  $x \neq y$  (they are alternatives)
- 2) let  $xP_jy$  and  $yP_kx$  ( $j \neq k, j, k \in I$ )
- 3)  $xPy$  and  $yPx$  by Condition L<sup>19</sup>

This inconsistency is not a consequence of liberal values but, rather, of Sen's characterization. As demonstrated above how can we say that the pair  $(x,y)$  as defined above be decisively private to both  $j$  and  $k$ ? We must specify, in accordance with the rules above, whose choice it is. Having done so the social decision function will produce one best choice according to the *libertarian standard*. Sen passed very quickly over this result in his original article; Sen (1970). No restatement or update was given in Sen (1976). We must surmise the proof from a few words.

---

<sup>18</sup> Sen (1970 p. 154). Note how illiberal the statement is.

<sup>19</sup> The inconsistency is not resolved for just two individuals. Just insert 1 and 2 for  $j$  and  $k$  and the same proof of inconsistency results. Only by reducing the number of individuals to one will the inconsistency dissolve.

Case I. Proof. Let the two individuals referred to in Condition L\* be 1 and 2, respectively, and the two pairs of alternatives referred to be (x,y) and (z,w), respectively. If (x,y) and (z,w) are the same pair of alternatives, then there is a contradiction.<sup>20</sup>

There is not a contradiction if both 1 and 2 prefer either x or y. Contradiction results when 1 prefers either x or y and 2 prefers the other as in the inconsistency proof above for Condition L itself. Is this proof of inconsistency relevant for decisive private choices? Here we have both 1 and 2 decisive with respect to the same pair (x,y). Either we are not, here, analyzing decisive private choices or the appropriate party with the decisive private choice must be identified. Presume that the pair  $x,y \in DPS_1$ , then the result is  $xPy$ . Otherwise the proof is irrelevant. It is, therefore, the one best choice given the libertarian claim. Note that Condition P had nothing to say about this outcome since it was not true that for all i either  $xP_iy$  or  $yP_ix$ .

Case III. Next, let x,y,z, and w, be all distinct. Let 1 prefer x to y, and 2 prefer z to w. And let everyone in the community including 1 and 2 prefer w to x and y to z. There is no contradiction for 1 and 2, for 1 simply prefers w to x, x to y and y to z, while 2 prefers y to z, z to w, and w to x. By Condition U this configuration must yield a social choice function. But by Condition L\* society should prefer x to y and z to w, while by the Pareto principal society must prefer w to x, and y to z. This means that there is no best alternative for the set, and a choice function does not exist for any set that contains these four alternatives."<sup>21</sup>

Here there are no shared elements of decisive choice. Presume that {x,y} and {w,z} are true elements of decisively private choice for j and k respectively. Otherwise the proof is irrelevant. We get two separate though not mutually exclusive outcomes. Sen asserts first that we do not get one best choice under the presumption that x and w are mutually exclusive social

---

<sup>20</sup> Sen (1970 p.154)

<sup>21</sup> Sen (1970 p.154)

states . The social decision function differs by four features , as Gibbard (1974) called them, and one best solution according to the libertarian claim emerges. The two mutually exclusive solutions are resolved at the individual level and the two mutually feasible solutions remain. For example, Paul sleeps on his back and Peter sleeps on his belly. Because each decisive private sphere is pairwise disjoint from every other, the analysis is easily extended from 1 and 2 to j and k. We may add Angela sleeps on her side, Peter reads *Reader s Digest*, etc. and there is still one best solution according to the libertarian claim. That claim is consistent. There is no unique solution according to Condition P. If Condition L is dropped from the analysis, we find no *social decision function* can simultaneously satisfy Conditions U and P for a quadruple.<sup>22</sup> The inconsistency comes from P, not L (as modified).

### III. Privately indecisive choices

The libertarian claim extends far beyond equation 1.4 ( $\forall i( x,y \in DPS_i \Rightarrow (xP_iy \Rightarrow xPy$  and  $yP_ix \Rightarrow yPx)$ ). Although there are a multitude of choices for which 1.4 may apply, there is an even larger set of choices for which mutual consent is required. Towers of sets may be constructed to reflect the possibilities for all the potential actions that constitute areas of exchange, in the broadest sense of the word, which apply. Interpersonal dependence, and therefore exchange, are pervasive features of a liberal society. Sen s example, *Lady Chatterly s Lover*, is as good a place to start as any.

whether you should sleep on your back or your belly is a matter which society should permit you absolute freedom, even if the community is *nosy* enough to feel you should sleep on your back. <sup>23</sup> (Italics added) It is this statement that forms the basis of the case of *Lady Chatterly s Lover* (LCL) which involves nosy people. He uses this to establish the notion of

---

<sup>22</sup> I leave this exercise to the reader. Hint: the answer is given in the proof in case III.

<sup>23</sup> Sen (1970 p.152)

*meddlesome preferences*. The example produces meddlesome preferences in both directions. Also the utility of one individual is dependent upon the disutility of the other, again in both directions. Unanimity requires that, if  $x$  is preferred to  $y$  by all, move to  $x$ . Can such a rule based on preferences alone, deal adequately with such preferences? If so, can it deal with these preferences in a liberal fashion, allowing each person to be decisive in some choices? The answer is yes to all these questions.

*Lady Chatterly's Lover* is an interesting play on words. Recall that we have Prude (1) and Lascivious (2), each with meddlesome preferences. Sen presents us with the following subset, three social states, from  $U$ .

$x = 1$  reads LCL  $y = 2$  reads LCL and  $z =$  no one reads LCL

Prude prefers  $z$  from  $\{x,z\}$  but prefers  $x$  from  $\{x,y\}$ . From  $\{x,y,z\}$  Prude orders the alternatives  $zP_xP_y$ . Lascivious prefers  $y$  from  $\{y,z\}$  but prefers  $x$  from  $\{x,y\}$ . From  $\{x,y,z\}$  Lascivious orders the alternatives  $xP_LyP_Lz$ . Hence, *if everyone else in society also prefers  $x$  to  $y$* , then we have  $xP_y$ .<sup>24</sup>

The *reason* that Prude would prefer to read the obscene book is that he does not want Lascivious to read it. Prude would *not* prefer  $x$  if it did not also mean that Lascivious would not be able to read it. Unstated in alternative  $x$  is the simultaneous requirement that Lascivious does not read it. So more accurately we would say that:

$x = 1$  reads LCL **and** 2 does not

To understand this from the perspective of our liberal values we need to establish the more primitive choices:

$r = 1$  reads it,  $s = 2$  reads it,  $t = 1$  does not read it,  $u = 2$  does not read it,  $x = r$  **and**  $u$ ,  $y = s$  **and**  $t$   
and  $z = t$  **and**  $u$

$y = 2$  reads LCL and 1 does not

$z = 1$  does not read LCL and 2 does not read LCL

The first rule of logic is that each statement be *atomistic*. It is necessary to first reduce each statement to a statement that cannot be further decomposed and then combine them through logical connectives into more complex propositions. This Sen has not done. He then claims that, by the libertarian claim, Prude is decisive over  $\{x, z\}$  and Lascivious is decisive over  $\{y, z\}$ . That is, Prude is decisive over whether (1 reads LCL and 2 does not) and (1 does not read LCL and 2 does not read LCL). Meanwhile, Lascivious is decisive over whether (2 reads LCL and 1 does not) and (1 does not read LCL and 2 does not read LCL).

Neither  $x, y$  nor  $z$  are elements of the decisive private sphere of Prude or Lascivious. Each element is potentially mutually exclusive with respect to the decisive private choice of another. Prude is not decisive over  $u$  and so he can not be decisive over  $x = r$  **and**  $u$ . Prude may be privately decisive with respect to the set  $\{r, t\}$ . (Prude may be decisive over  $r =$  Prude reads LCL and  $t =$  Prude does not read LCL) Lascivious may be privately decisive with respect to  $\{s, u\}$ . (Lascivious may be decisive over  $s =$  Lascivious reads LCL and  $u =$  Lascivious does not read LCL) The best Prude can do *decisively* is to choose from  $\{r, t\}$ . The best Lascivious can do *decisively* is to choose from  $\{s, u\}$ . If another option is available, feasible and preferable they are free to exchange. Individuals at liberty to do so will if the transaction costs are not prohibitive and there is a mutually beneficial exchange. The third problem with Condition L is now exposed. There is not a one to one correspondence between preference, *with respect to a single pair*, and choice. This is where  $(xP_i y \Rightarrow xP y)$  and  $(yP_i x \Rightarrow yP x)$  breaks down. The libertarian claim is exactly, no more and no less, that:

$$(1.10) D_j(x, y) \Leftrightarrow x, y \in DPS_j$$

---

<sup>24</sup> Note the level of unanimity required here. Everyone else is nosy and with the same preference as Lascivious.

$D_j$  means the option to choose decisively. A decisive choice of  $j$  from the elements  $x, y$  implies that all of those elements are in the decisive private sphere of  $j$ . If all the elements of choice are in the decisive private sphere then  $j$  *may* choose decisively from those elements. Or equivalently,

$$(1.11) D_j(x,y) \text{ if and only if } x,y \in \text{DPS}_j$$

The condition that  $x$  and  $y$  are elements of the decisive private sphere of  $j$  is the necessary and sufficient condition that he *may* choose decisively from that group. It is not a prohibition against him choosing otherwise.

The decisive private sphere is not a prohibition to *interpersonal* exchange of elements of their decisive private spheres. This is quite common. Each individual may be decisive over where he lives, but often may agree to move under sufficient compensation. An individual may not be decisive over  $x = \text{live in Wyoming}$  and  $y = \text{work for IBM}$ , though he is decisive over  $\{x,z\}$  (where  $z = \text{live in New York}$ ). IBM may not be decisive over that pair  $\{x,y\}$ . If John prefers to live in Wyoming they cannot compel him to work for IBM in New York. John may certainly choose  $w = \text{live in New York and work for IBM}$ , if IBM consents and the compensation is sufficient. The libertarian prohibition is against *social alienation* of these elements. Society may not delete the right to choose where he lives from John's elements of decisive private choice. The concept of the *protected sphere* is discussed fully in subsequent sections. Sen claims the social decision function *must* choose from their decisive private spheres (which he got wrong anyway) rather than their mutually preferred choice (which  $x$  is).

Condition MC: (mutual consent) For all  $j, k$ , if  $x \in \text{DPS}_j$  and  $y \in \text{DPS}_k$  and  $w = x$  **and**  $y$ ,

$$\text{then } C_j(\cdot) = w \Leftrightarrow C_k(\cdot) = w$$

The individual  $j$  may choose a compound including an element of the decisive private sphere of  $k$ , if and only if  $k$  chooses likewise. (Presuming that coercion is not lurking in the background.)

Neither does this choice by Prude and Lascivious need to be preferable by all others.

The unanimity is only local to these two participants to exchange. Note that this choice is also not mutually exclusive with respect to the privately decisive choices of all other individuals.

Every other individual may still decide whether to read or not read Lady Chatterly's Lover, sleep on their back, etc. Therefore, the choice by Prude and Lascivious from elements of their decisive private spheres is also binding on society. Symbolically,

$$C_P(\{r,s,t,u\}) = C_L(\{r,s,t,u\}) = \{r \text{ and } u\} = x \Rightarrow xP(r,s,t,u)$$

Case II of the impossibility proof can now be addressed as well.

They have therefore at most one alternative in common, say  $x = z$ . Assume now that person 1 prefers  $x$  to  $y$  and person 2 prefers  $w$  to  $z$  ( $=x$ ). And let everyone in the community including 1 and 2 prefer  $y$  to  $w$ . There is no inconsistency for anyone, not even for 1 and 2, and their respective orderings are: 1 prefers  $x$  to  $y$  and  $y$  to  $w$ , while 2 prefers  $y$  to  $w$  and  $w$  to  $x$ . By Condition U this should be in the domain of the social decision making. But by Condition L\*,  $x$  must be preferred to  $y$  and  $w$  must be preferred to  $x$  ( $=z$ ), while by the Pareto principal,  $y$  must be preferred to  $w$ . Thus there is no best element in the set  $(x=z,y,w)$  in terms of social preference, and every alternative is worse than some other. A choice function for society does not therefore exist. <sup>25</sup>

In case two, each of the individuals share an element of decisive choice. Since  $j$  is decisive over  $\{x,y\}$  and  $k$  is decisive over  $\{x,z\}$  we can get  $xP_jy$  and  $zP_kx$ . Again, either we are not discussing decisively private choices or the inconsistency is irrelevant. If  $x,y \in DPS_j$ , then  $k$ 's preferences with respect to  $x$  are irrelevant. If  $z \in DPS_k$ , then  $k$  needs another decisively private element to have a decisive choice. This is case III discussed above. If  $x \in DPS_k$ , then  $j$ 's preference with respect to  $x$  is irrelevant. Again, if  $x$  is John sleeps on his back, then  $x$  is not an

element of the decisive private sphere of Karl. If x is a compound of elements from their respective private spheres, mutual consent is required.

In *Lady Chatterly s Lover*, any of the compound elements x, y or z requires the unanimous consent of the parties involved. Other examples in the literature exhibit this same problem of taking a compound of decisively private elements and demonstrating an inconsistency. The next topic, then, is *Gibbard s paradox*. This paradox pertains to a conformist and a non-conformist whose preferences cannot, presumably, be reconciled.

Sen uses the example of Zubeida and Rehana. Zubeida wants to dress like Rehana and Rehana wishes to differentiate herself from Zubeida. Here the compounds are  $rr =$  Zubeida wears red **and** Rehana wears red or  $rg =$  Zubeida wears red **and** Rehana wears green.<sup>26</sup> Neither of these are elements of decisive private choice for either of them. It would require mutual consent. The only way to grant either of them the right is to limit the liberty of the other and impose on the other an obligation. If it is accepted that the way the person dresses is a person s own business and *whatever* she decides about her own dressing must be judged to be socially better, then there is now a problem of consistency.<sup>27</sup> (Italics added) This statement has nothing to do with liberal thought. *Whatever*, the key word here, does not entitle Zubeida to order Rehana s life or vice versa. That is a *violation* of libertarian principals. All we have here is another play on words or as Blau remarked, The whole situation has a liberal ring.<sup>28</sup> Neither Zubeida nor Rehana may be given the liberty to compel the other to dress as *they* please.

---

<sup>25</sup> Sen (1970 p. 154)

<sup>26</sup> There are two other choices  $gg$  (both wear green) and  $gr$  (green and red). This is often put into a non-cooperative game theory framework, an entirely inappropriate model. These two parties are not locked in separate rooms being interrogated by the district attorney. Co-operative game theory would indicate they would contract over the problem if a mutually satisfactory solution is available.

<sup>27</sup> Sen (1976 p. 234)

<sup>28</sup> Blau (1975. p. 367)

It is also because they have decisive private spheres that a mutually acceptable result can emerge. Neither may seek to coerce the other by appeals to the social planner or collective decision rule to decide whose preferences should control, i.e. to make an interpersonal comparison of utility. Should Rehana offer to wash Zubeida's car if she will wear green and Zubeida accept, then Rehana will have a right with respect to Zubeida and Zubeida will have an obligation to Rehana. Rehana and Zubeida may also determine which direction money compensation must flow to accommodate their differing preferences. At some point one becomes a seller and the other a buyer. These are the appropriate methods to reconcile their preferences.

Their true preferences will be revealed whether or not an exchange takes place. If a welfare enhancing exchange is feasible, given the transactions cost and their preferences, they will exchange or contract. If not, they will not exchange and each will have to live with the outcome from their decisive spheres. This provides the incentives for the exchange. Suppose exchange does not occur. Their preferences cannot be reconciled. The required compensation to induce cooperation from one party exceeds the benefit to the other. The demonstration that *unsatisfied preferences* may exist is neither a remarkable insight, nor inconsistent with the concept of Pareto optimality or libertarian values. No social decision function will leave no preferences unsatisfied. Every economist knows this.

This is a good point to stop and reflect on the concept of 'one best solution'. Sen claims: If more than one person is given the right to make a class of personal decisions without outside interference, this itself can give rise to a preference cycle.<sup>29</sup> This implication is, of course, backwards. The preference cycle exists prior to the private decisions. The question is how will the preference cycles be resolved. The libertarian solution is voluntary exchange. The social decision function that requires one best solution, on a social basis, would require that we make

an interpersonal comparison for every feasible preference set with respect to even the most personal aspects of our individual lives. The one best solution requires that John sleep on his back, Peter sleep on his belly, Angela marries Peter, Rehana and Zubeida both wear red, etc as decided by a *social standard*. Every aspect of every life must be conformed to that one best solution, however it is determined. We have seen that theory in action. Our cherished liberal values find that concept totally abhorrent.

Another class of decisions is private but never decisively so. Take the simple but pervasive matter of choice in marriage. Let  $x$  = Angela marries John and  $y$  = Caroline marries John. These are not compound in the sense above where two decisively private choices are combined. The act of marriage requires the onerous action of another by the nature of the act.<sup>30</sup> It therefore requires their consent. Neither Angela, John nor Caroline is decisively positioned with respect to this choice. The following statement is not feasible. Each individual can marry whomever they choose and this right applies to all. From the simple example above it is clear that Angela and Caroline cannot both have the right to marry whomever they choose; John. Angela's right to marry John would imply an obligation on John to marry her and no like right for Caroline. The marriage of Angela to John is mutually exclusive with respect to Caroline and John. Gibbard (1974) tried to shoehorn this type of choice into Sen's assigned pair approach and from there he found he needed alienable rights.<sup>31</sup> Kelly (1976) proclaimed the virtues of Gibbard's approach. Basu (1992) demonstrated the futility of that approach. The entire discussion is not relevant for matters that require mutual consent. In such cases, the maximum liberty consistent with the like liberty of another is just the freedom to choose and act

---

<sup>29</sup> Sen (1976 p. 234). This was the example above.

<sup>30</sup> deJasay, Anthony (1996 p. 5-31). deJasay has written an exceptionable book. The concepts of liberties, rights and obligations are dealt with by clear and direct thought. This short book is a brilliant treatment of liberal concepts.

indecisively. The libertarian rule is that each individual is at liberty to marry whomever consents. Each individual exercises a matching liberty through mutual consent.<sup>32</sup>

Irrespective of the fact that each individual is highly unique and therefore a differentiated product, the market that develops is highly competitive. The outcome of that market will be pairings of marriage partners and the result of that process will be in accordance with their individual, constrained preferences. The alternatives are extremely illiberal. The social planner can order each marriage according to a social standard that differs from the individual standard. Suppose he decides that John should marry Angela instead. All of the other pairings will vary also. Caroline may or may not even get her second choice. *In the absence of coercion*, every single marriage pairing would revert to the individual standard. Society, whatever that means, would be better off but what could we say about the welfare of the individuals in that society?

Alternatively, a collective decision-rule could be used to make the decision of who should marry whom. Nothing approaching unanimity would be feasible and information available to the voters about pairings would be extremely sparse. The outcome would be socially desirable according to the standard that it complied with the collective decision rule. I cannot imagine the desirable properties this outcome would have other than it satisfied the decision rule. Again, the outcome would not be optimal by an individual standard. In the absence of coercion, every single marriage pairing would revert to the individual standard. The collective would be better off, according to the collective standard.

---

<sup>31</sup> Gibbard also initiated the assigned pair language in the literature. He sometimes spoke as if it was a trump card. You can have at least one pair, which you select depending on the situation. Alienable rights were also introduced by Gibbard.

<sup>32</sup> deJasay (1996 p. 5-31)

#### IV. Social choices and the Paretian epidemic

In this section focus shifts from private to public choices and from the private to the protected sphere. The concept of the private sphere is connected to the concept of the protected sphere by the following rule.

$$(1.12) \forall i(f_i: x,y, z \rightarrow DPS_i) \Rightarrow x,y, z \notin SC$$

where SC is the set of elements from which social choices are made.

In words, if the libertarian rule sends elements to the decisive private sphere of each individual, those elements are not elements of the domain of social choice. This *liberal partition* is another essential element of the libertarian claim. Equivalently, the union of each decisive private sphere intersects the domain of social choice nowhere.

$$(1.13) \cup_i DPS_i \cap SC = \emptyset$$

This is not to say that the State has no interest in the individual liberties of those within it. The enforcement role of the State is a necessary condition of the libertarian regime. This is usually accomplished by the enforcement of a non-arbitrary rule of law. The rule of law establishes, in advance, the limits of individual freedom as it impinges upon the maximum liberty consistent with the like liberty of all others. Criminal, tort, nuisance, property and contract law established over centuries of dealing with actions which tested the limits of freedom gives us a body of common law which is quite comprehensive. The fact that there is an enforcement role does not, however, entitle society to include in their *social choice* elements from the decisive private spheres of those within it. Constitutional democracies have absolute prohibitions against interference with the private sphere of individuals. This is the concept of the *protected sphere*. This is all very basic material but material which Sen seems to overlook.

This liberal partition is a two way street. It protects the individual private sphere from interference by the social decision function but it is also designed to protect the domain of social choice from interference by the individual. That is,

$$(1.11) \quad x, y, z \in SC \Rightarrow x, y, z \notin \cup_i DPS_i$$

Sen's Paretian epidemic follows, again, from the failure to distinguish those elements of choice for which an individual may be decisive and those for which he may not. Recall from Section II, Sen's standard for the relevance of Condition L. He asserts that if the choices were all non-personal, Condition L would not be relevant. By that standard Condition L would be relevant for the following choice:

$x$  = Angela paints her walls blue and  $y$  = build an interstate highway system.

Although  $y$  is non-personal,  $x$  is personal so this pair might be assigned to Angela by the not all non-personal standard. This pair is clearly not a decisively private choice for Angela by the libertarian rule. Neither should this pair be assigned to Angela by the *some* element of private choice standard, i.e. one of the elements of choice is private. By the liberal partition, Angela's preferences with respect to the color of her walls are not to be aggregated into the social decision function at all. Consequently, she would not be *potentially semi-decisive* over all pairs of social choice under a libertarian structure. We saw in the review of Sen's proof that individuals would only be decisive over those pairs in their decisive private spheres. They are *potentially semi-decisive* over those pairs for which exchange is feasible through mutual consent. They are decisive over the pairs they should be decisive over. They are *potentially semi-decisive* over the pairs they should be *potentially semi-decisive*. They have a voice in collective decisions but they are not *potentially semi-decisive* by virtue of their individual liberties. All this while liberty abounds across a wide range of actions.

## V. Summary

The failure to specify those pairs for which each individual may be privately decisive produced numerous logical errors in the impossibility result presented by Sen (1970) and extended to the Paretian epidemic in Sen (1975). The existential claim (Condition L) was subjected to a universal test (Condition U) in conjunction with Condition P. Had Condition P not been in the analysis the obvious connection between an existential claim and a universal test would have been immediately apparent. The failure to decompose each statement and use logical connectives in the examples, together with the failure to specify those elements, for which the individuals may chose decisively, led to additional logical errors.

No one to date has commented on tenuous connection between Condition P and the concept of Pareto optimality or the relevance of that standard for decisively private choices. This paper has not either. That rich topic requires another paper. Like Sugden (1985), this paper has focused on the logic of liberty. Rowley (1993) warned against the ability of an axiomatic approach to capture all the nuances of social theory. Sen has repeatedly asserted that while his Condition L\* may not capture all the nuances, it is essential to any more elaborately detailed claim. This paper demonstrates, quite clearly, the error in that statement.

The existence of the rule,  $f_L$ , which provided that the decisive private sphere for each individual was not empty, provides only *minimal liberalism*. ( $\forall i(DPS_i \neq \emptyset)$ ) The expansion of liberty through Condition MFL enlarges the scope of individual liberty to include those actions which each individual may take which are not mutually exclusive with the like liberty of others. This does require the *onerous forbearance* of all.<sup>33</sup> John may prefer that Karl sleep on his back rather than his belly. Karl may therefore be harmed by John's choice. The harm incurred by Karl presumes that John's existence is justified only by serving Karl's will. I find this anti-Kantian

notion a morally abhorrent concept. Whether you find that morally abhorrent or not, consequentially, no *action* by anyone will pass the test of no harm to another by this standard. It is most certain that no decision by a social standard would pass that test. It is not a feasible standard.

Condition MC further expands that liberty under the constraint of mutual consent. Many new actions are available to the individual that requires the onerous action of another but only by their mutual consent. There are classes of action which by their nature require mutual consent. The absence of coercion and the presence mutual consent are absolutely necessary to the optimal properties that result from production and exchange. Coercion distorts the opportunity cost of one of the parties, making him choose differently than if he were at liberty to select from the options feasibly available. The decisive private sphere, together with the requirement of mutual consent, enables the parties to engage in exchange from that uncoerced condition to a mutually beneficial state. The fact that their feasible sets are enlarged through cooperation provides the incentive to do so. Neither may presume the other exists to do their will but must discover welfare enhancing exchange through mutual consent. Vast new actions become available to all under this arrangement.

The question remains, then, whether there are actions for which a social standard is relevant. The examples of marriage, how people sleep, etc. were intended to demonstrate the irrelevance of a social standard with respect to these types of choices. Buchanan (1954) questioned why Arrow thought that social rationality could be made consistent with individual values. They cannot. Social rationality requires a social standard. What optimal properties would the social standard produce and under what circumstances? These are the important questions.

---

<sup>33</sup> deJasay (1996 p.5-31)

Coincidence with the collective decision-rule is not an optimal property. It is compliance with the rule.

Given our liberal values, the more important question is which aggregate preferences, judged by a social standard, can be exercised in light of individual liberties. Arrow was addressing the question of whether there could be a social welfare function in light of individual preferences. The debate was framed in terms of unconstrained individual preferences and without any regard for individual liberties. Hence the title of this paper. Social choice and individual liberties is a research program deserving of significant attention.

## References

- Arrow, Kenneth J. A Difficulty in the Concept of Social Choice. *Journal of Political Economy* LVII (1950): 328-46
- Basu, Kaushik. The Right to Give Up Rights. *Economica* 51 (November, 1984): 413-22
- Blau, Julian. Liberal Values and Independence. *Review of Economic Studies* XLII (1975): 395-401
- Buchanan, James M. Social Choice, Democracy, and Free Markets. *Journal of Political Economy* LXII (1954): 114-23
- deJasay, Anthony. *Before Resorting to Politics*. The Shaftsbury Papers, 5. Series Editor: Charles K. Rowley. Cheltenham, U.K. and Brookfield, U.S.: Edward Elgar Publishing, 1996
- Farrell, M.J. Liberalism in the Theory of Social Choice. *Review of Economic Studies* XLIII (1976): 3-10
- Gibbard, Allan. A Pareto — Consistent Libertarian Claim. *Journal of Economic Theory* 7 (1974): 388 - 410
- Rowley, Charles K., eds. *Social Choice Theory I*. Brookfield, Vermont: Edward Elgar Publishing Company, 1993.
- Schelling, Tom. *Conflict and Organization*. Cambridge, Mass: Harvard University Press, 1984
- Sen, Amartya. The Impossibility of a Paretian Liberal. *Journal of Political Economy* 78 (1970): 152-7
- Sen, Amartya. Liberty, Unanimity and Rights. *Economica* 43 (171) (1976): 218-38
- Sowell, Thomas. *Knowledge and Decisions*. New York, NY: Basic Books Publishers Inc., 1980
- Sugden, Robert. Social Choice and Individual Liberties. *Economics and Philosophy* 1 (1985): 213-29